Prevention of dissipation with two particles

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Abstract

An error prevention procedure based on two-particle encoding is proposed for protecting an arbitrary unknown quantum state from dissipation, such as phase damping and amplitude damping. The schemes, which exhibits manifestation of the quantum Zeno effect, is effective whether quantum bits are decohered independently or cooperatively. We derive the working condition of the scheme and argue that this procedure has feasible practical implementation.

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Decoherence and loss limit the practicality of quantum cryptography and computing [1,2]. To circumvent this difficulty, successful quantum error correction techniques have been developed [3-15]. In these schemes, some redundancy is introduced to protect the original information. If mere a few qubits are subject to errors in a noisy environment, these errors can be corrected by detecting the error syndromes, without violation of the original information. On the one hand, quantum error correction holds the promise of reliable storage, processing, and transfer of quantum information; on the other hand, it is rather costly of computing resources. For correcting single-qubit errors, one needs at least five qubits to encode one qubit information [7,8]. The encoding, decoding, and detection of error syndromes are also involved.

Compared with the conventional error correction schemes, alternate decoherence-reducing strategies based on the quantum Zeno effect are more efficient to implement, though they may fail for some models of decoherence[16,17]. The use of the Zeno effect for correcting or for preventing errors in quantum computers was first suggested by Zurek [18], and it is a part of a scheme considered by Barenco *et al.* [19]. Recently, Vaidman *et al.* proposed an error prevention scheme of this kind based on four-particle encoding [20]. It is shown there that four is the minimal number of particles required for prevention of general errors. Nevertheless, if one has more knowledge about the errors, simpler codes can be found. It is discovered that two qubits are enough for preventing pure dephasing due to phase damping [21,20], or for preventing pure loss due to balanced amplitude damping to a reservoir at absolute zero temperature [22,23]. In the error correction schemes mentioned above, different qubits are assumed to be decohered independently. This is an ideal case. As another ideal case, if two adjacent qubits are decohered completely collectively, there exists an alternate error prevention

scheme based on two-particle encoding which utilizes the coherence-preserving states of qubit-pairs [24].

In this paper, we propose an error prevention scheme based on two-particle encoding for reducing a large range of decoherence. The scheme exhibits manifestation of the quantum Zeno effect. Compared with the known error prevention schemes, this scheme has two favorable features. First, it covers a large range of decoherence. The scheme is designed for preventing phase damping and general amplitude damping. The reservoir may be at arbitrary temperature, and the damping coefficients need not be balanced among different qubits. Furthermore, the scheme works whether the qubits are decohered independently or cooperatively. Second, it has a high efficiency. Two qubits are enough for encoding one qubit. The encoding, decoding, and error-detection in this scheme are quite simple. We also extend the scheme for protecting states of multiple qubits. It is shown that approximately $L+\frac{1}{2}\log_2\left(\frac{\pi L}{2}\right)$ qubits are enough for encoding L qubit. The scheme costs less computing resources, so it is easier to be implemented in practice.

First, we look at two qubits, which are subject to decoherence described by phase damping or amplitude damping to a reservoir at arbitrary temperature . The qubits are described by Pauli's operators $\overrightarrow{\sigma}_l$ $(l=1,2)$ and the reservoir is modelled by a bath of oscillators. The bath modes coupling to the l qubit are denoted by $a_{\omega l}$ (ω varies from 0 to ∞ and $l=1,2$). Some of the modes $a_{\omega 1}$ and $a_{\omega 2}$ are possibly the same and some of them are different. We use the notation $\bigcup\limits_{l=1}^{2} a_{\omega l}^{+}a_{\omega l}$ to indicate the joint sum of $a_{\omega l}^{+}a_{\omega l}$, so $\bigcup\limits_{l=1}^{2} a_{\omega l}^{+}a_{\omega l} = a_{\omega 1}^{+}a_{\omega 1} + a_{\omega 2}^{+}a_{\omega 2}$ if $a_{\omega 1}$ and $a_{\omega 2}$ belong to different modes and $\bigcup\limits_{l=1}^{2} a_{\omega l}^{+}a_{\omega l} = a_{\omega 1}^{+}a_{\omega 1}$ if $a_{\omega 1}$ and $a_{\omega 2}$ are the same. With this notation, the Hamiltonian describing dissipation of the two

qubits has the following form (setting $\hbar = 1$)

$$H = \omega_0 \left(\sigma_1^z + \sigma_2^z\right) + \sum_{l=1}^{2} \int d\omega \left[g_{\omega l} A_l \left(a_{\omega l}^{+} + a_{\omega l}\right)\right] + \int d\omega \bigcup_{l=1}^{2} \left(\omega a_{\omega l}^{+} a_{\omega l}\right), \tag{1}$$

where the coping coefficients $g_{\omega l}$ may be dependent of ω and l. The qubit operator A_l in general is expressed as a linear superposition of three Pauli's operators, i.e., $A_l = \lambda^{(1)}\sigma_l^x + \lambda^{(2)}\sigma_l^y + \lambda^{(3)}\sigma_l^z$. The ratio $\lambda^{(1)} : \lambda^{(2)} : \lambda^{(3)}$ is determined by the type of the dissipation. For example, $\lambda^{(1)} = \lambda^{(2)} = 0$ for phase damping and $\lambda^{(3)} = 0$ for amplitude damping [25]. Phase damping induces pure dephasing and amplitude damping induces loss and dephasing at the same time. Many sources of decoherence in quantum computers can be described by amplitude damping [26].

The qubit operator A_l satisfies the condition $tr\left(A_l\right) = 0$, so without loss of generality , its two eigenvectors are denoted by $\left|\pm 1\right\rangle_l$, with the eigenvalues ± 1, respectively. The physical basis vectors $\left|\pm\right\rangle_l$ are eigenstates of the operator σ_l^z. The states $\left|\pm 1\right\rangle_l$ in general differ from $\left|\pm\right\rangle_l$ by a single-qubit rotation operation $R_l\left(\theta\right)$, i.e., $\left|\pm 1\right\rangle_l = R_l\left(\theta\right)\left|\pm\right\rangle_l$, where θ depends on the type of the dissipation. The computation basis vectors $\left|0\right\rangle_l$ and $\left|1\right\rangle_l$ in this paper are defined by

$$\begin{aligned} \left|0\right\rangle_l &= \tfrac{1}{\sqrt{2}}\left(\left|+1\right\rangle_l + \left|-1\right\rangle_l\right), \\ \left|1\right\rangle_l &= \tfrac{1}{\sqrt{2}}\left(\left|+1\right\rangle_l - \left|-1\right\rangle_l\right). \end{aligned} \tag{2}$$

The are derived from $\left|\pm 1\right\rangle_l$ by a Hadamard transformation H_l. The states $\left|0\right\rangle_l$ and $\left|1\right\rangle_l$ have the important property that they are flipped by the qubit operator A_l. In general, the computation basis and the physical basis differ by a single-qubit rotation operation. But if $\lambda^{(3)} = 0$, i.e., for amplitude damping, these two bases reduce to the same.

We use the two dissipative qubits to protect one qubit information. The initial state of one qubit (the information carrier) is generally expressed as $\left|\Psi\left(0\right)\right\rangle_1 =$

$c_+ \left|+\right\rangle_1 + c_- \left|-\right\rangle_1$. The other qubit (the ancilla) is prearranged in the state $\left|+\right\rangle_2$. We use the symbol C_{ij} to denote the quantum controlled NOT (CNOT) operation in the physical basis, where the first subscript of C_{ij} refers to the control bit and the second to the target. The input state of the information carrier is encoded into the following state

$$\left|\Psi\left(0\right)\right\rangle_1 \otimes \left|+\right\rangle_2 \xrightarrow{C_{12}R_1(\theta)R_2(\theta)H_1H_2} \left|\Psi_{enc}\right\rangle = c_+ \left|01\right\rangle + c_- \left|10\right\rangle. \tag{3}$$

For amplitude damping, the above joint operation $C_{12}R_1\left(\theta\right)R_2\left(\theta\right)H_1H_2$ reduces to a simple CNOT C_{12} in the physical basis. But for more general dissipation, this encoding requires the knowledge of the noise parameters $\lambda^{(1)} : \lambda^{(2)} : \lambda^{(3)}$. The encoding space spanned by the sates $\left|01\right\rangle$ and $\left|10\right\rangle$ is denoted by the symbol S_0, which is a subspace of the whole 2×2 dimensional Hilbert space of the two qubits.

For pure amplitude damping, states in the encoding space S_0 are left unchanged by the free Hamiltonian $H_0 = \omega_0\left(\sigma_1^z + \sigma_2^z\right)$ of the qubits. But for more general dissipation, the Hamiltonian H_0 may map the initial encoded state out of the encoding space. This probability should be avoided. The free-Hamiltonian-elimination (FHE) technique is introduced to attain this goal. We apply a homogeneous classical far-violet-detuned optical field E to the two qubits. Under the adiabatic approximation, the additional Hamiltonian describing the driving process reads [27]

$$H_{drv} = -\sum_{l=1}^{2} \frac{2\left|g\right|^2\left|E\right|^2}{\omega_{opt} - \omega_0}\sigma_l^z, \tag{4}$$

where ω_{opt} is the frequency of the optical field. By adjusting the intensity $\left|E\right|^2$ of the optical field, we choose the coefficient in Eq. (4) to satisfy $\frac{2\left|g\right|^2\left|E\right|^2}{\omega_{opt}-\omega_0} = \omega_0$. The effect of the free Hamiltonian H_0 is thus offset by the driving field. Compared with the FHE procedure involved in Ref. [24], this technique has the advantage

that it operates without the knowledge of the noise parameters.

Our scheme is based on the quantum Zeno effect. The protection procedure consists of frequent tests that the two-qubit system has not left the encoding space. Now we show that an arbitrary encoded state is indeed frozen through these tests. The initial density operator of the reservoir is denoted by $\rho_r(0)$. Suppose $|\Psi_{ru}(0)\rangle$ is a purification of $\rho_r(0)$, i.e., the state $|\Psi_{ru}(0)\rangle$ satisfies $tr_u(|\Psi_{ru}(0)\rangle\langle\Psi_{ru}(0)|) = \rho_r(0)$, where the symbol u denotes an ancillary system. During a finite time T_0, we perform N times tests. In a short period of time T_0/N, under the Hamiltonian (1) and (4), the encoded state (3) evolves into

$$\begin{aligned}|\Psi(t)\rangle &\approx [1 - i(H + H_{drv}) T_0/N] |\Psi_{enc}\rangle \otimes |\Psi_{ru}(0)\rangle \\ &= |\Psi_{enc}\rangle \otimes \left[1 - \tfrac{1}{N}\int d\omega \bigcup_{l=1}^{2} \left(i\omega T_0 a_{\omega l}^{+} a_{\omega l}\right)\right] |\Psi_{ru}(0)\rangle \qquad (5) \\ &-\tfrac{1}{N}\left[|00\rangle \otimes (c_+X_2 + c_-X_1)|\Psi_{ru}(0)\rangle + |11\rangle \otimes (c_+X_1 + c_-X_2)|\Psi_{ru}(0)\rangle\right],\end{aligned}$$

where $X_l = \int d\omega \left[ig_{\omega l}T_0\left(a_{\omega l}^{+} + a_{\omega l}\right)\right]$. This evolution has two important properties: First, after the evolution the amplitude of the state outside the encoding space is of the order of $1/N$. Hence if we perform a measurement in succession to tell us whether the two-qubit system has left the encoding space S_0, the probability for getting the result "out of S_0" is of the order of $1/N^2$. Second, after the evolution the amplitude of the state inside the encoding space remains the same as the initial encoded state. Therefore, if we get the result " in S_0" in the measurement, the encoded state is unchanged by the dissipation. The two-qubit system after the test without a postselection of the measurement results is in a mixed state, whose density operator is represented by $\rho(T_0/N)$. The operator $\rho(T_0/N)$ can be expressed as

$$\rho(T_0/N) = \hat{S}(T_0/N)\,\rho(0), \qquad (6)$$

where $\rho(0) = |\Psi_{enc}\rangle\langle\Psi_{enc}|$ and $\hat{S}(T_0/N)$ is a superoperator. Obviously $\hat{S}(T_0/N)$

has the following decomposition

$$\hat{S}\left(T_0/N\right) = \hat{I} + O\left(1/N^2\right), \tag{7}$$

where $\hat{I}$ is the unit superoperator. After the time T_0, the final state $\rho\left(T_0\right)$ of the qubits is thus

$$\rho\left(T_0\right) = \left[\hat{S}\left(T_0/N\right)\right]^N \rho\left(0\right) = \left|\Psi_{enc}\right\rangle \left\langle\Psi_{enc}\right| + O\left(1/N\right). \tag{8}$$

This equation suggests that the difference between the final and the initial state of the qubits is of the order of $1/N$ and can be neglected for large N. After decoding the final state, we successfully protect one qubit information from decoherence by the two dissipative qubits.

The remaining question is how to test that the two qubits has not left the encoding space. The states $|0\rangle_l$ and $|1\rangle_l$ are orthogonal to each other, so they are eigenstates of an observable B_l. We perform a quantum non-demolition (QND) measurement of the observable B_1+B_2 on the two qubits. The states of the qubits lie in the encoding space if and only if the measurement of B_1+B_2 yields the result 1. The QND measurement of B_1+B_2 can be done in the following way: First prepare an ancilla qubit 3 in the state $|0\rangle_3$ and then apply a joint operation $C_{13}'C_{23}'$ to the three qubits, where $C_{ij}' = R_i\left(\theta\right) R_j\left(\theta\right) H_i H_j C_{ij} H_i H_j R_i\left(-\theta\right) R_j\left(-\theta\right)$ is the quantum CNOT operation in the computation basis $\left\{|01\rangle, |10\rangle\right\}$. By testing whether the state of the ancilla qubit 3 has been flipped, we perform a QND measurement of the observable B_1+B_2.

Eq. (8) is approximate result, which is obtained under the condition of large N. An important question is then how frequently we should perform the tests. After each period of time, with some probability we get a wrong state. This probability P_{err} is measured by the norm of the amplitude of the state outside

the encoding space. Suppose the reservoir is initially in thermal equilibrium. Following Eq. (5), P_{err} can be expressed as $P_{err} = \frac{\delta}{N^2}$, where

$$\delta = \sum_{l=1}^{2} \int d\omega \left[|g_{\omega l}|^2 T_0^2 \coth\left(\frac{\hbar\omega}{2k_B T}\right)\right] + 4Re\left(c_+^* c_-\right) \int d\omega \left[Re\left(g_{\omega 1}^* g_{\omega 2}\right) T_0^2 \coth\left(\frac{\hbar\omega}{2k_B T}\right)\right]$$
$$\leq \int d\omega \left[\left(|g_{\omega 1}| + |g_{\omega 2}|\right)^2 T_0^2 \coth\left(\frac{\hbar\omega}{2k_B T}\right)\right]. \tag{9}$$

After the time T_0, the accumulated error rate P_{tot} is estimated by

$$P_{tot} \approx N P_{err} = \frac{\delta}{N}. \tag{10}$$

Therefore, the scheme successfully prevents errors only when $N >> \delta$, where the magnitude of δ is determined by the damping coefficients $|g_{\omega l}|^2$, the evolution time T_0, and the temperature T of the reservoir. Eq. (10) suggests, a larger N, a smaller accumulated error rate. However, this is not the case in practice, since we need also consider the unavoidable noise introduced by the frequent tests. Suppose γ is the additional error rate introduced by the test after each period of time. We assume that γ satisfies $\gamma << \delta$. With this realistic consideration, the accumulate error rate is rewritten as

$$P_{tot} \approx \frac{\delta}{N} + N\gamma \geq 2\sqrt{\delta\gamma}. \tag{11}$$

The minimum error rate is achieved if N equals $\sqrt{\frac{\delta}{\gamma}}$, which is the optimal value of the times of the tests. The error rate γ introduced by every time test should be very small so that $2\sqrt{\delta\gamma} << 1$. This serves as the working condition of our scheme.

The above scheme can be extended to include multiple qubits. If we have $2L$ dissipative qubits, of course they can be exploited to protect L qubits information by encoding and detecting every two qubits in the way described above. The efficiency η of the scheme is $\frac{1}{2}$. This efficiency can be further raised. For the

two-qubit circumstance, we notice that the encoding space is an eigenspace of the operator $B_1 + B_2$. Similarly, for the $2L$-qubit circumstance, the eigenspace of the operator $B_1 + B_2 + \cdots + B_{2L}$ can also be used as the encoding space. An arbitrary state in this eigenspace is mapped into another eigenspace by each of the operator A_l. So the first-order error caused by the coupling to the reservoir can be detected and prevented by measuring the observable $B_1 + B_2 + \cdots + B_{2L}$. The largest eigenspace of the operator $B_1 + B_2 + \cdots + B_{2L}$ has a dimension $\left(\begin{array}{c} 2L \\ L \end{array} \right)$, with the eigenvalue 0. The maximum efficiency $\eta_{\max}$ thus attains

$$\eta_{\max} = \frac{1}{2L} \log_2 \left(\begin{array}{c} 2L \\ L \end{array} \right) \approx 1 - \frac{1}{4L} \log_2 (\pi L), \tag{12}$$

where the approximation is taken under the condition of large L. The efficiency $\eta_{\max}$ is near to 1 for large L.

We have shown that two qubits subject to noise described by the Hamiltonian (1) are enough for protecting one qubit information. The Hamiltonian (1) is quite general. Its different special case yields the coupling for amplitude damping or for phase damping. Our scheme works whether the qubits couple independently to separate environments or cooperatively to the same environment. However, it is important to examine the type of the noise beyond our description. The scheme relies on the Zeno effect, so it can deal only with "slow" noise. The error should not accumulate too quickly [16]. The quantum Zeno effect dose not take place when the time interval between the tests is larger than the characteristic time for which the exponential decay approximation is applicable, therefore noise can be prevented only when error occurs at a sub-exponential rate [23]. These limitations are also suffered by other error-prevention schemes based on the Zeno effect [20,23].

The most favorable feature of the present scheme is that it prevents error by

costing few computing resources. This feature is remarkable since the quantum computing resources are very stringent [28-30]. The proposed error prevention code is very simple, so it has a good chance to be implemented in experiments or to be used in numerical simulations of the robustness of quantum information.

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